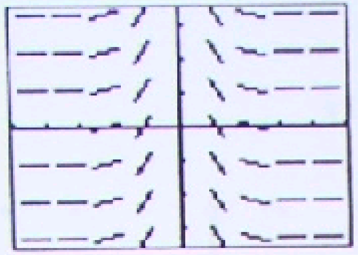
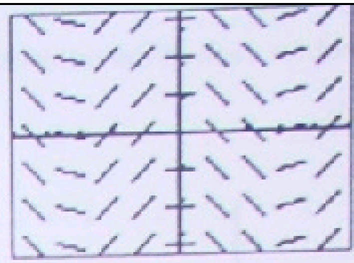
$y=\frac{1}{x^2}+c$

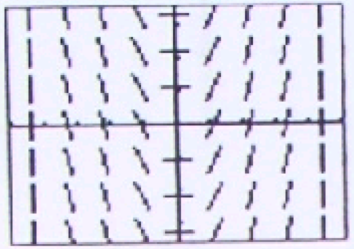




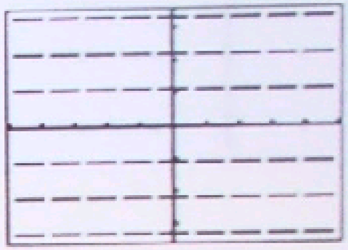
$y=\cos x+c$

(C)

$y=x^2+c$



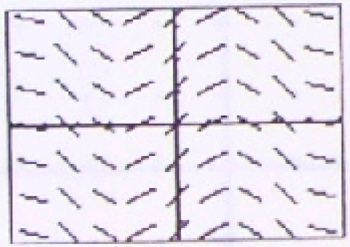
(D)



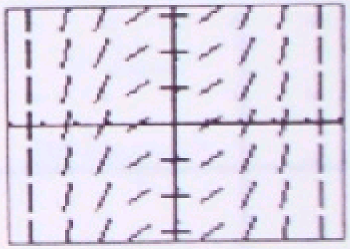
$y=1+c$

(E)

$y=\sin x$

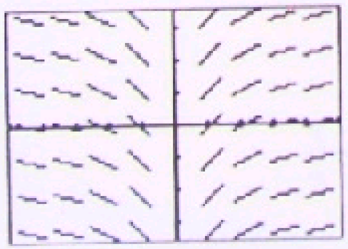


(F)



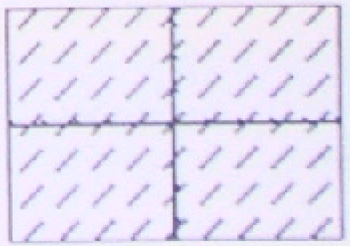
$y=\frac{1}{6}x^3$

(G)



$y=\ln|x|$

(H)



$y=x$

Trigonometric Formulas

$$\int \cos u\,du = \sin u + C \qquad \int \sin u\,du = -\cos u + C$$

$$\int \sec^2 u\,du = \tan u + C \qquad \int \csc^2 u\,du = -\cot u + C$$

$$\int \sec u \tan u\,du = \sec u + C \qquad \int \csc u \cot u\,du = -\csc u + C$$

Exponential and Logarithmic Formulas

$$\int e^u\,du = e^u + C \qquad \int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\frac{\partial}{\partial x} 3^x = 3^x \ln 3$$

$$\int \ln u\,du = u\ln u - u + C \quad \text{(See Example 2)}$$

$$\int \log_a u\,du = \int \frac{\ln u}{\ln a}\,du = \frac{u\ln u - u}{\ln a} + C$$

EXAMPLE 2 Verifying Antiderivative Formulas

Verify the antiderivative formulas:

(a) $\int u^{-1}\,du = \int \frac{1}{u}\,du = \ln|u| + C$ (b) $\int \ln u\,du = u\ln u - u + C$

SOLUTION

We can verify antiderivative formulas by differentiating.

(a) For $u > 0$, we have $\frac{d}{du}(\ln|u| + C) = \frac{d}{du}(\ln u + C) = \frac{1}{u} + 0 = \frac{1}{u}$.

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represent the general antiderivative of f, but in fact it is quite useful for *dealing with the effects of the Chain Rule* when function composition is involved. Exploration 1 will show you why this is an important consideration.

EXPLORATION 1 Are $\int f(u)\,du$ and $\int f(u)\,dx$ the Same Thing?

Let $u = x^2$ and let $f(u) = u^3$.

1. Find $\int f(u)\,du$ as a function of u. $\int u^3\,du = \frac{u^4}{4} + C$
2. Use your answer to question 1 to write $\int f(u)\,du$ as a function of x. $\frac{(x^2)^4}{4} + C = \frac{x^8}{4} + C$
3. Show that $f(u) = x^6$ and find $\int f(u)\,dx$ as a function of x. $\int f(u)\,dx$ $\int x^6\,dx = \frac{x^7}{7} + C$
4. Are the answers to questions 2 and 3 the same?

Exploration 1 shows that the notation $\int f(u)$ is not sufficient to describe an antiderivative when u is a function of another variable. Just as du/du is different from du/dx when differentiating, $\int f(u)\,du$ is different from $\int f(u)\,dx$ when antidifferentiating. We will use this fact to our advantage in the next section, where the importance of "dx" or "du" in the integral expression will become even more apparent.

EXAMPLE 3 Paying Attention to the Differential

$) = x^3 + 1$ and let $u = x^2$. Find each of the following antiderivatives in terms of x:

- Read Sect. 6.2
- Quick Review
- Do 6.2 #1–16 $\left(\frac{1}{2}\right)$
  #17–24 $\left(\frac{1}{2}\right)$